

# Questions for the Graphics, Animation, Rendering, Modeling Qualifier

Spring 2013

## GENERAL QUESTIONS

- 1) **Physically correct stick motion:** This is a 2D problem. Consider a stick of constant length  $L$  and uniform thickness and density. You are given the initial positions  $(A_0, B_0)$  and a final positions  $(A_1, B_1)$  of the two endpoints. Provide the high-level description and then the geometric details for rendering a physically plausible animation that interpolates these two keyframes. *Hint: Explain how to compute the end positions  $(A_t, B_t)$  for time  $t$  assuming that the stick is not subject to any external force.*
- 2) **Steady stick motion:** This is a 2D problem. Consider a stick of constant length  $L$  and uniform thickness and density. You are given the initial positions  $(A_0, B_0)$  and a final positions  $(A_1, B_1)$  of the two endpoints. Provide the high-level description and then the geometric details for rendering a steady animation that interpolates these two keyframes. *Hint: Explain how to compute the end positions  $(A_t, B_t)$  for time  $t$  assuming that each point of the stick moves at constant speed along a circular path.*
- 3) **Drawing on a triangle mesh:** This is a 3D problem. The user wants to be able to draw curves on a triangle mesh by drawing the strokes with the mouse on the screen over a perspective view of the mesh. Then these strokes should be “baked” onto the mesh (as if really painted on the mesh) and displayed in other views. Explain how you would implement this functionality without using a texture map or image projection, because you want to be able to draw extremely high precision curves, in case the user moves the viewpoint close to the mesh. *Hint: You may assume that the mesh is represented using a Corner Table or similar data structure and that you have a function, which, given the current mouse location, returns the 3D surface point visible through the corresponding pixel.*
- 4) **Interpolation:** You are told that the motion  $P(t)$  of a point in 3D is subject to a constant, although unknown acceleration (which needs not be vertical). (a) You observe 3 consecutive positions:  $P(0)=A$ ,  $P(1)=B$ ,  $P(2)=C$ . Provide the simplest possible expression of  $P(3)=D$  in terms of  $A$ ,  $B$ , and  $C$  and prove its correctness. (b) You observe 3 positions:  $P(a)=A$ ,  $P(b)=B$ ,  $P(c)=C$ . Provide the details of a simple geometric construction or algorithm for computing of  $P(d)=D$  in terms of  $A$ ,  $B$ ,  $C$ ,  $a$ ,  $b$ ,  $c$ , and  $d$  and prove its correctness. *Hint: You may be able to derive the answers from first principles and basic math. Strive to provide a concise answer and an elegant formulation.*
- 5) **Tangle:** You are given two polygonal loops in 3D  $A$  and  $B$ , with vertices  $A_1 \dots A_a$  and  $B_1 \dots B_b$ , respectively. (Each is a closed-loop, manifold, and planar polygon boundary, but  $A$  and  $B$  are not coplanar.) Provide a simple algorithm for testing whether  $A$  and  $B$  are entangled (i.e., cannot be smoothly deformed and separated without collision) and justify its correctness. Comment on its optimality (its computational complexity versus the best possible one). *Hint: You may assume that curves  $A$  and  $B$  have no point in common.*
- 6) **Mirror:** Your eye  $E$  is at distance  $d$  from a window through which you see a room with 3 points  $A$ ,  $B$ ,  $C$  and a perfect mirror in which you see their reflections  $A'$ ,  $B'$  and  $C'$ . You don't know where the mirror is. Let  $a$ ,  $b$ ,  $c$ ,  $a'$ ,  $b'$ , and  $c'$  be points on the window through which you see these 3D points. Can you compute the normal  $N$  to the mirror from  $d$ ,  $a$ ,  $b$ ,  $c$ ,  $a'$ ,  $b'$ , and  $c'$ ? If yes, then provide the construction (the simpler and more elegant—the better) and its justification. If not, explain why not and what information is missing. *Hint: You need not provide the detail of standard geometric constructions, such as computing a circumscribing circle or the intersections of simple curves.*

# ANIMATION

## 1) Fluid simulation:

Consider a 2D fluid simulation using MAC grid to represent the pressure and velocity fields. Figure 1 indicates the current state of the fluid, where blue, gray, and white blocks denote fluid, solid and air respectively.

- Please describe how to use pressure projection to enforce boundary conditions and fluid incompressibility.
- If there is a small piece of solid in the fluid, how is two-way coupling enforced (Figure 2)?
- Why is the projection matrix always symmetric positive definite?

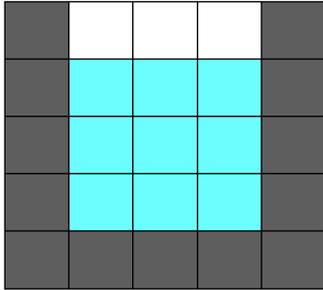


Figure 1

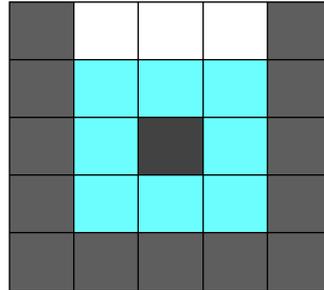


Figure 2

## 2) Closed-loop dynamics:

Consider an articulated rigid body system with 8 links described in the generalized coordinates (Figure 3).

- If we define a constraint to connect the last and the first links, how do we compute the constraint force in generalized coordinates (Figure 4)?
- When the system is in contact with the floor, how do we solve the constraint force in addition to the contact forces (Figure 5)?

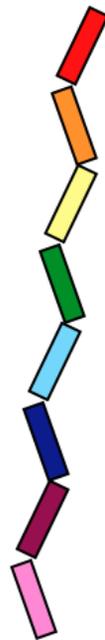


Figure 3

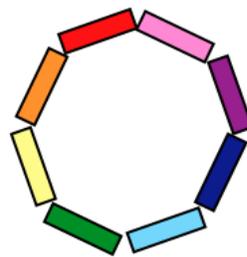


Figure 4

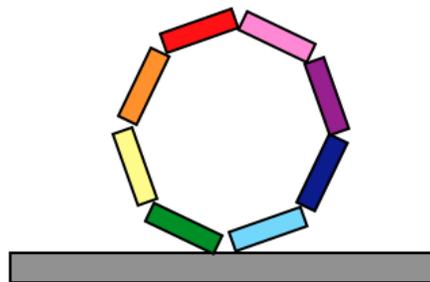
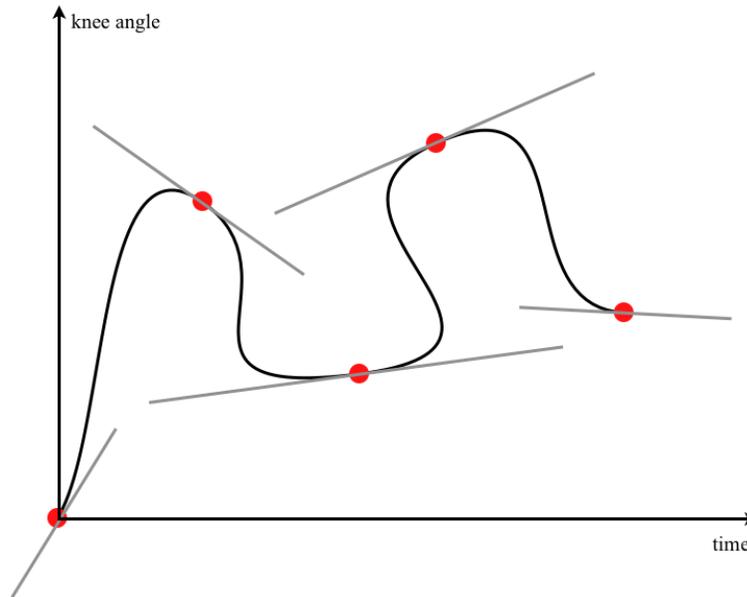


Figure 5

3) **Interpolation:**

An animator tried to use the “pen tool” in Adobe Illustrator to interpolate a set of key poses for the “knee joint”. Each key is represented as a point  $(x, y)$  in a 2D domain, where x-axis indicates the time and the y-axis indicates the knee joint angle. Once a set of keys is defined, the animator used “pen tool” to control the shape of the curve connecting these keys by defining the tangent vector at each point. However, the animator wished to create a curve that is a function of time,  $y = f(x)$ , such that each time instance only maps to one joint value. Can you come up with such a function with minimal changes to the curve the animator designed?



4) **Inverse dynamics:**

Consider a biped character made of articulated rigid bodies represented in the generalized coordinates. Assuming both feet are in contact with the floor and there are no joint limits, torque limits, or contact force constraints, can we always achieve the desired acceleration for all the degrees of freedom (DOFs) without applying illegal torques to the three translational and three rotational DOFs at the root? If your answer is yes, please prove it mathematically. If your answer is no, please provide a counter example.

## RENDERING

1. **Soft Effects for Film:** You are working at an animation studio, and your project manager has put you in charge of writing a renderer for the next film. Your renderer must be able to achieve three specific soft effects: penumbra, glossy reflections, and motion blur. Your project manager has also stated that this must be done using a rasterization renderer, *without* using ray tracing. How will you achieve these three effects in the same spirit as distributed ray tracing (sampling a probability distribution), but using only rasterization? What is the computational cost of your approach? What are the potential visual artifacts of your approach, and what can you do to minimize them?
2. **Pen-and-Ink Silhouettes:** You have decided to compare making pen-and-ink style silhouettes using two different input formats. One input is an animated 3D polygon mesh. The other is a sequence of images taken from a video. In seeking the highest quality results, you know that coherence between the frames of the animation is of major importance. Describe how you would create high-quality silhouette animations from each of these two kinds of input. Give enough detail that another graduate student would know how to proceed.
3. **Wood Grain Textures:** You have been given the task of rendering polygonal mesh sculptures as if they were carved from wood. The particular wood grain you are supposed to simulate has very distinct light and dark bands. You already have found out that these bands can be the source of considerable problems with aliasing. Describe the details of two possible solutions to rendering this wood grain with minimal aliasing under each of these conditions: 1) using an image texture that has been mapped to the mesh, and 2) with a Perlin-style solid texture that has been described as a procedural texture.
4. **Titanic II:** James Cameron has begun making a film of the Titanic II, a ship designed after the original doomed vessel and that also sank. The film has many indoor scenes where global illumination effects are important. Describe a rendering system that will produce high quality global illumination effects, but at a computational cost that is not prohibitive. Your solution should work for scenes that have a large amount of geometric detail. After describing your approach, indicate its most likely computational bottlenecks and the factors that influence this cost.